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Screened-Coulomb ansatz for the nonfactorizable radiative corrections to off-shell W^+W^- production

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Abstract. We demonstrate that the results of the complete first-order calculation of the nonfactorizable QED corrections to the single-inclusive cross sections for $e^+e^- \to W^+W^- \to 4$ fermions can be reproduced by a simple, physically motivated ansatz. The ansatz allows us to effectively take into account the screening role of the non-Coulomb radiative mechanisms by introducing a dampening factor in front of the width-dependent part of the known first-order Coulomb correction, the so-called screened-Coulomb ansatz.

1 Introduction

A precise study of W boson physics is one of the main objectives of the LEP2 program; a future high-energy electron (muon) collider will open up unique new possibilities. This physics goal requires very accurate theoretical knowledge of the standard model predictions for the process

$$e^+e^- \to W^+W^- \to 4 \text{ fermions}. \tag{1}$$

In particular, the role of QED radiative corrections, as well as that of finite-width effects, should be understood in detail [1].

It is well known that the instability of W bosons (the W-boson width $\Gamma_W \approx 2.1\,\mathrm{GeV}$) can strongly modify the "stable W" results. Special attention should be paid to the radiative interferences (both virtual and real) that interconnect the production and decay stages of the process (1). In particular, there is a class of contributions corresponding to the so-called "charged-particle poles" [1–4] that may induce a strong dependence of differential distributions on W-boson virtualities. The final-state interactions may result in nonfactorizable QED radiative corrections to the Born cross section of (1). Recall that the level of suppression of the width-induced effects depends on the degree of inclusiveness of the distribution. Thus, for the totally inclusive cross section, the QED nonfactorizable corrections cancel up to the terms of $\mathcal{O}(\alpha\Gamma_W/M_W)$ [2,3]. In contrast, differential distributions could be distorted on the level of $\mathcal{O}(\alpha)$. Particular attention should be paid to the threshold region,

$$E = \sqrt{s} - 2M_W \sim \mathcal{O}(\Gamma_W). \tag{2}$$

Here the instability-induced modification of the Coulomb interaction between the slowly moving W bosons is especially significant (for details, see [2,5]). In [5] it is shown that the W-boson width effects drastically change the on-shell value of the Coulomb correction, even at $E \gg \Gamma_W$, but that after integration over the invariant masses of the W bosons, the result corresponding to the stable W bosons is recovered far above the threshold region.

Recall that in the threshold region, the Coulomb contribution can be uniquely separated from the other electroweak corrections. In the relativistic region, however, it is neither uniquely defined nor gauge-invariant. At larger W-boson velocities β, the width-induced modifications of the differential distributions caused by other radiative mechanisms (for example, intermediate–final or final–final state interferences [2,4]) may become just as important. These mechanisms may contribute to both factorizable and nonfactorizable corrections. It is discussed in [4] that in the relativistic region, a cancellation between the different sources of instability takes place. As a result, the nonfactorizable corrections may vanish and the stable-W result may be recovered. In the ultra-relativistic limit, $(1-\beta) \ll 1$, such a cancellation appears quite naturally; in fact, it has its origin in the conservation of "charged" currents (see, e.g., [6–8]).

In the intermediate region, $\beta \lesssim 1$, which is relevant for the current LEP2 energy range, an analysis of the nonfactorizable corrections to the differential cross sections of W^+W^- production requires detailed study.

During the last few years there has been a significant progress in our understanding of the radiative effects in the off-shell gauge-boson pair production [1]. In particular, recently a complete calculation of the nonfactorizable corrections to the process (1) has been independently performed by two groups [9–11] (see also [4]). The results are quite consistent with each other.

In [8], an attempt has been made to estimate the possible screening impact of other radiative interference mechanisms on the Coulomb scenario in the relativistic regime.

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Using physical intuition, the authors briefly consider the consequences of the so-called dampened (screened) Coulomb ansatz. Within this ansatz, an extra screening factor $(1-\beta)^2$ is introduced in front of the width-dependent arctan term in the known first-order unstable Coulomb formula (see, e.g., [5,12]). Such a simple prescription is motivated by a model analysis of [4], where a simple scenario is considered: one of the W bosons is assumed to be stable. Another example of the cancellation of the off-shell effects at relativistic energies has been known for quite a while (see, e.g., [6,13]). When considering the gluon radiation corresponding to the top production and decay at very high energies, one observes that the width-dependent effects vanish when the emission at the production and decay stages are added.

Two obvious advantages of the screened-Coulomb ansatz are that it is very simple and that it readily allows for a transparent physical interpretation. It could be useful from the point of view of practical applications as well. It will require, however, special detailed study in order to understand whether this scenario can be taken as a realistic plausible baseline.

We aim in this paper to perform a detailed comparison of the screened-Coulomb ansatz with the results of the recent calculations [9–11]. It appears that this simple prescription provides a surprisingly reasonable quantitative understanding of the screening of the nonfactorizable terms at higher energies. The results of this paper can be applied equally well to the $\gamma\gamma$- initiated processes.

The paper is organized as follows. In Sect. 2 some basic formulas are presented. In Sect. 3 we study numerically several characteristic observables. We conclude in Sect. 4. The appendix contains a quantitative analysis of the screening effects based on the explicit Feynman diagram calculations.

2 Ansatz for the nonfactorizable corrections

In the Born approximation for the process (1) there are three (signal) diagrams where two resonant W bosons are produced and the background diagrams where, at most, one resonant W boson is formed. The background diagrams are typically suppressed by $\mathcal{O}(\Gamma_W/M_W)$ $[\mathcal{O}(\Gamma_W^2/M_W^2)]$ with respect to the leading double resonant contributions.

The currently most favourable approach adopted for the calculation of the radiative corrections to the processes involving unstable particles is the so-called *pole scheme* [14]. In the double-pole approximation, one considers the complete off-shell process as a superposition of the production of a pair of unstable particles and their subsequent decays. The radiative effects are then naturally separated into two groups: factorizable and nonfactorizable. The first type includes radiative corrections which can be unambiguously attributed to either the production or the decay stage of the process; they exhibit simple analytical behaviour in the limit $\Gamma_W \to 0$. The second type corresponds to the radiative interconnections between various stages of the process.

It is instructive to trace the physical origin of such separation at a point not far from the threshold. When considering soft photons, $k^0 = \omega \ll M_W$, the production and decay of the W bosons can be regarded essentially as pointlike processes with a characteristic time scale $t_{\mathrm{char}} \sim 1/M_W$. However, due to the W decays, various stages are separated in time by an intervals $\tau \sim 1/\Gamma_W$. When we average over the times between W-pair production and the W decays, a significant interconnection occurs only in the $\omega \lesssim \Gamma_W$ domain. This results in the nonfactorizable correction. The contribution to these corrections caused by the hard photons is power-suppressed (see, e.g., [2,13, 15]).

When examining the process (1) one distinguishes three energy domains:

- Threshold region (2) where Ws are moving with a small velocity with respect to each other, $\beta \sim \sqrt{\Gamma_W/M_W} \ll 1$.
- Non-relativistic region, $\Gamma_W \ll E \ll M_W$, where the velocity of the Ws is still a small parameter, $\beta \ll 1$, but the center-of-mass energy is sufficiently far from threshold.
- Relativistic region, $E \sim M_W$, where velocity of the Ws is not a small parameter any more, $\beta \sim 1$.

Recall that in the threshold and the nonrelativistic region, the main contribution to the radiative corrections comes from the Coulomb interaction (see [5,12]). All other effects are suppressed by $\mathcal{O}(\beta)$. Near threshold, the Coulomb contribution dominates the instability effects. In the relativistic region, the terms suppressed in the nonrelativistic region are not small, and should be taken into account. The explicit calculation of the complete nonfactorizable correction performed in [9] uses the "far from threshold" (FFT) approximation, which assumes that $\Gamma_W \ll E$. The accuracy of this approximation is $\mathcal{O}(\Gamma_W/E)$. This approximation breaks down in the threshold region, but it is valid in the (far-from-threshold) nonrelativistic region and in the relativistic region. Note that in the nonrelativistic region, the calculation of the complete nonfactorizable correction agrees with the calculation of the off-shell Coulomb effect within the adopted approximations.

We discuss below a simple ansatz based on the Coulomb result (screened-Coulomb ansatz) which appears to be in good agreement with the complete calculation of the nonfactorizable corrections in both the relativistic and nonrelativistic regions. Of course, one cannot expect that a simple unique prescription exists that would allow one to reproduce reasonably well the results of the explicit complete calculations of the nonfactorizable corrections to the arbitrary differential distribution[1]. Below we concentrate

[1] When considering the angular distributions of the final-state fermions, the general arguments based on the conservation of the charged currents (see, e.g., [6]) may be not applicable, and the screened-Coulomb ansatz could be irrelevant. The largest discrepancies should be observed near the edges of the kinematic phase space, where the corrections are the largest but the event statistics very limited. A detailed study of the dependence of the nonfactorizable corrections on the fermion angles has been presented in [9–11].

on the quantities, which are inclusive with respect to all the decay and production angles, such as the invariant mass spectrum of a W boson.

Since the calculation of the nonfactorizable corrections in [9–11] has been performed in the FFT approximation, we shall remain within the same scheme for the screened-Coulomb ansatz. This means that we shall not consider the threshold region here. The reader is reminded that the latter region has been studied in detail elsewhere (see, e.g., [5,12]). For reference purposes, we consider also a model case where the cross section is corrected by the Coulomb effect only. Then the differential distribution over an observable X can be written in the following form[2]

$$\begin{aligned} \frac{d\sigma_{\rm Coul}}{{\rm d}X} &= \frac{d\sigma_{\rm Born}}{{\rm d}X}\left(1+\delta_{\rm Coul}\right), \\ \delta_{\rm Coul} &= \delta^{\text{on-shell}}_{\rm Coul} + \delta^{\rm nf}_{\rm Coul} \qquad (3) \\ \delta^{\text{on-shell}}_{\rm Coul} &= \frac{\alpha\pi}{2\beta}, \\ \delta^{\rm nf}_{\rm Coul} &= -\frac{\alpha}{\beta}\arctan\left(\frac{M_1^2+M_2^2-2M_W^2}{2M_W\Gamma_W}\right), \qquad (4) \end{aligned}$$

where M_1 and M_2 are the invariant masses of the W bosons, β is their on-shell velocity,

$$\beta = \sqrt{1-4M_W^2/s}, \qquad (5)$$

and ${\rm d}\sigma_{\rm Born}/{\rm d}X$ is the on-shell Born cross section. This is the leading contribution to the radiative correction in the nonrelativistic region. All other contributions, which were neglected, are suppressed by $\mathcal{O}(\beta, \Gamma_W/E)$, at least. Let us emphasize that outside of the threshold region, the Coulomb approach is just an oversimplified extreme and, naturally, is not supposed to correspond to the true physics. Note that throughout this paper the so-called fixed-width scheme is used, where Γ_W is the on-shell W-boson width.

The two terms in (4) are of a different nature. The first one represents the factorizable part of the Coulomb interaction; it is completely the same as the familiar Coulomb effect for the stable case. It should be noted again that at high energies, where β is not a small parameter, this correction is of the same order as the rest of the radiative corrections, and is not enhanced in any way. Typically, the leading contribution coming from radiative corrections goes from $\sim \alpha\pi/\beta$, at threshold, to $\sim \alpha/\pi$ far from threshold.

The second term is the nonfactorizable part of the Coulomb correction. It arises due to the instability effects. It averages to zero when integrated over the invariant masses. As discussed in [2,3] (see also [5]), this is a general feature of the nonfactorizable corrections.

The physical reason for the separation between the factorizable and nonfactorizable corrections is rooted in the difference in the characteristic energies and momenta of the photons responsible for the different terms in (4).

In order to gain more insight into the above, let us consider the diagram with the photon exchange between the two W bosons. The denominator of the propagator of the W boson with the 4-momentum p_1^μ is

$$k^2 + 2kp_1 + D_1, \quad D_1 = p_1^2 - M_W^2 + {\rm i}\Gamma_W M_W. \qquad (6)$$

Not too far from threshold for the on-shell (factorizable) part of the Coulomb effect photons with energies $\omega \sim \beta^2 M_W$ and momenta $|\mathbf{k}| \sim \beta M_W$ are essential. It is worth-while to recall that $1/(\beta^2 M_W)$ is the typical interaction time between the W bosons, see [5]. In such a case k^2 can not be neglected in the W-boson propagator, contrary to the $\Gamma_W M_W$ term, see [2,5]. Therefore, the Coulomb effect here remains unchanged by the instability of the W bosons. On the other hand, only the photons with the energies $\omega \sim \Gamma_W$ and momenta $|\mathbf{k}| \sim \Gamma_W/\beta$ give the leading contribution to the off-shell part of the Coulomb effect. Note that β/Γ_W is the typical spatial separation between the diverging W bosons [12]. Far from threshold, at $M_W \gg E \gg \Gamma_W$, the two regions in the photon energy-momentum space are well separated. Because of this fact the effects are additive. Near threshold, where $E \sim \Gamma_W$, the two regions start to overlap, which is precisely the reason why our approach to the calculation of the double pole residues becomes invalid.

As has already been mentioned, in the relativistic domain, Coulomb correction does not account correctly for all the effects. Instead, complete nonfactorizable corrections are required

$$\frac{{\rm d}\sigma_{\rm nf}}{{\rm d}X} = \frac{{\rm d}\sigma_{\rm Born}}{{\rm d}X}\left(1+\delta_{\rm nf}\right). \qquad (7)$$

The explicit expressions for $\delta_{\rm nf}$ [9–11] are rather lengthy, and for the purposes of this paper there is no need to present them here.

Motivated by [4,8], we would like to check whether the complete nonfactorizable corrections could be approximated reasonably well by a simple ansatz based on the screening of the nonfactorizable (off-shell) part of the Coulomb effect

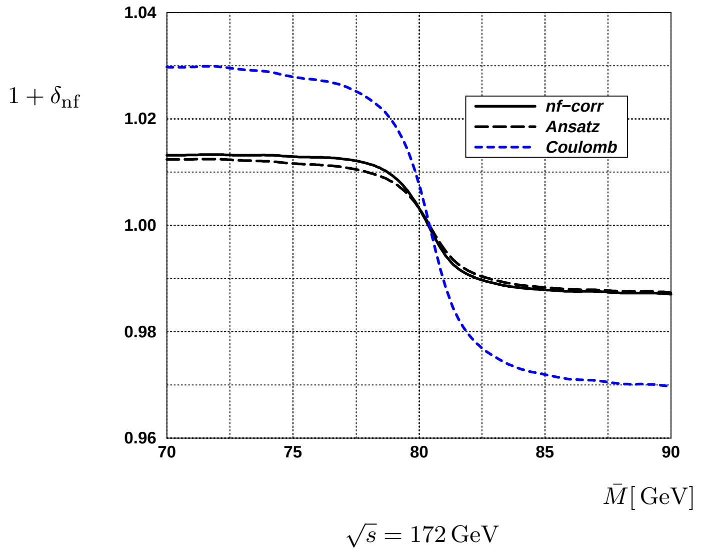
$$\frac{{\rm d}\sigma_{\rm Ans}}{{\rm d}X} = \frac{{\rm d}\sigma_{\rm Born}}{{\rm d}X}\left(1+\delta_{\rm Ans}\right), \qquad (8)$$

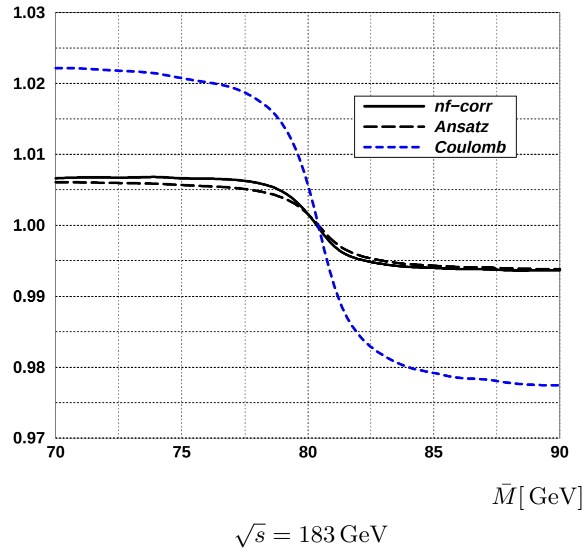
where

$$\delta_{\rm Ans} = \delta^{\rm nf}_{\rm Coul}(1-\beta)^2. \qquad (9)$$

Nonfactorizable corrections distort the Breit–Wigner distribution over the invariant mass of the W boson. This results, in particular, in the shift of the maximum of the invariant mass distribution. The potential importance of this effect is quite transparent, since such a shift may affect the measurement of the mass of the W boson. It is possible to estimate this shift from the relative nonfactorizable correction to invariant mass distribution. We will consider specifically the distribution over the average invariant mass $\bar{M} = (M_1+M_2)/2$. The standard expression

[2] Here and in what follows, we consider only the first-order Coulomb formulas. As shown in [8,16], the higher order Coulomb effects are practically negligible.





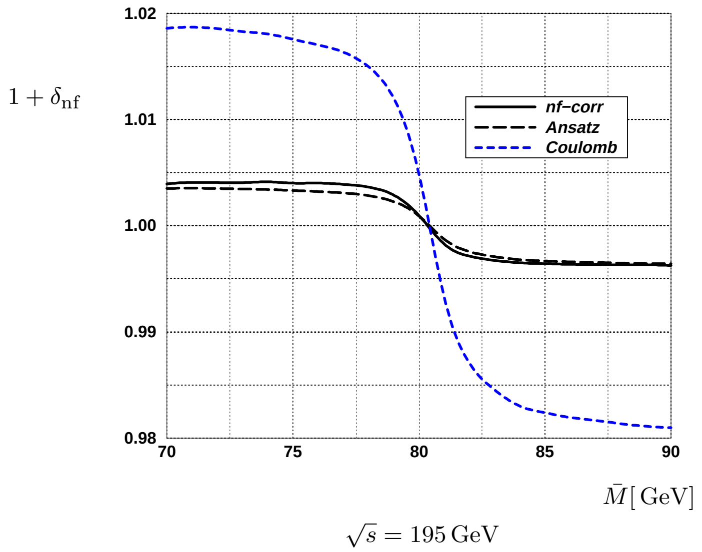


Fig. 1. The complete nonfactorizable correction to the distribution over the average invariant mass $\bar{M} = (M_1 + M_2)/2$ at $\sqrt{s} = 172$, 183 and 195 GeV, as compared to the expectations from the screened ansatz and from the unscreened-Coulomb scenarios

for the linearized shift is

$$\Delta\bar{M} = \frac{1}{8}\Gamma_W^2 \frac{\mathrm{d}\delta_{\rm nf}(\bar{M})}{\mathrm{d}\bar{M}}\Big|_{\bar{M}=M_W}. \qquad (10)$$

Based on the ansatz prescription (8) and (9) for the nonfactorizable correction, we arrive at the very simple formula for the shift (see also [4])

$$\Delta\bar{M} = -\frac{\alpha}{4}\frac{(1-\beta)^2}{\beta}\Gamma_W. \qquad (11)$$

In the following section we shall investigate numerically how this ansatz approximates the complete nonfactorizable correction to the single-inclusive distributions at various energies. In all cases, good agreement is established. We will show some specific examples which illustrate this statement.

3 Numerical results

In the following calculations, we assume

$$\begin{aligned} &\alpha = 1/137.0359895, \quad \alpha(M_Z) = 1/127.9, \\ &\sin^2\theta_W = 0.223, \\ &M_W = 80.41\,\mathrm{GeV}, \qquad \Gamma_W = 2.06\,\mathrm{GeV}, \\ &M_Z = 91.187\,\mathrm{GeV}, \qquad \Gamma_Z = 2.49\,\mathrm{GeV}. \end{aligned} \qquad (12)$$

We use $\alpha(M_Z)$ to calculate the Born cross sections, and α to calculate the radiative corrections. Results are presented in the LEP2 energy range $\sqrt{s} = 160-200$ GeV and for three discrete energies: $\sqrt{s} = 172$, 183, and 195 GeV. Several comparisons are made between the results of the complete calculation of the nonfactorizable corrections [9], the expectations based on the screened-Coulomb ansatz, and the model unscreened-Coulomb prescription. The latter scenario can help one to assess the impact of the non-

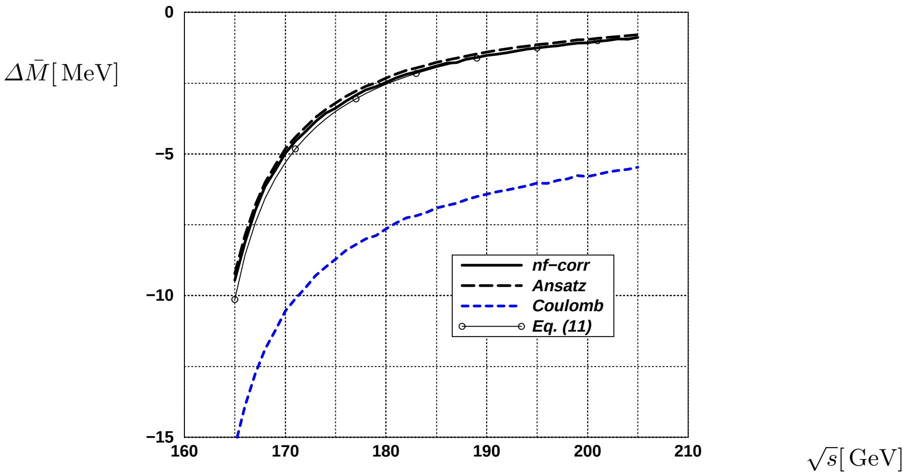
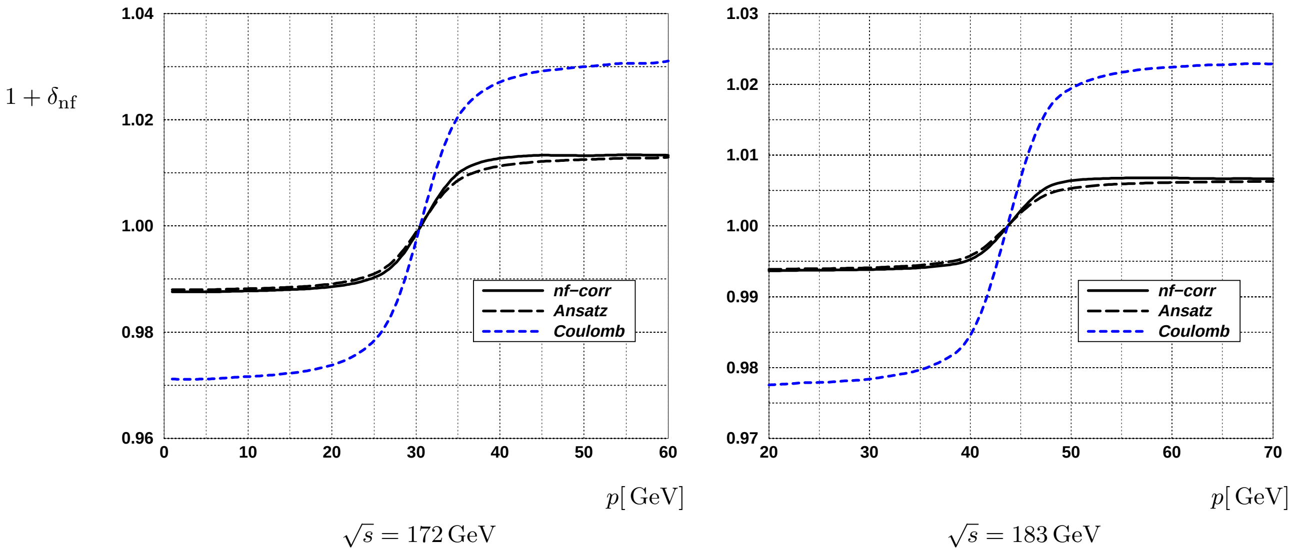


Fig. 2. The additional shift of the maximum in the distribution of the average invariant mass of two W bosons due to the nonfactorizable correction as a function of the collider energy, as compared to the expectations from the screened ansatz and from the unscreened-Coulomb scenarios



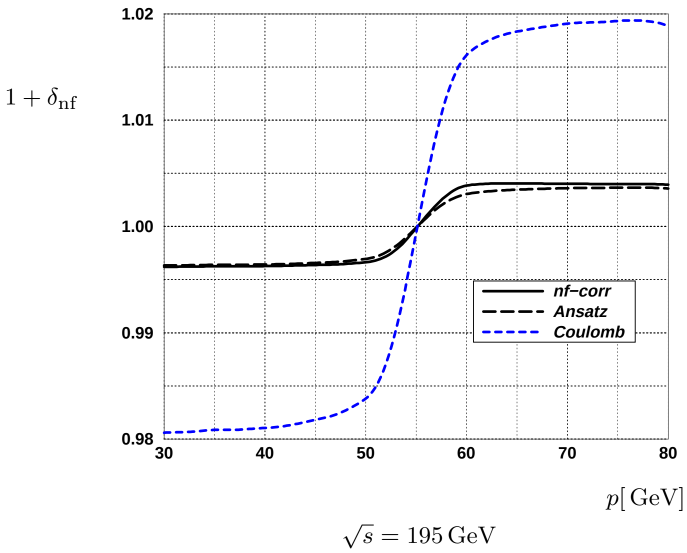


Fig. 3. The complete nonfactorizable correction to the distribution over the W momentum at $\sqrt{s}$ = 172, 183 and 195 GeV, as compared to the expectations from the screened ansatz and from the unscreened-Coulomb scenarios

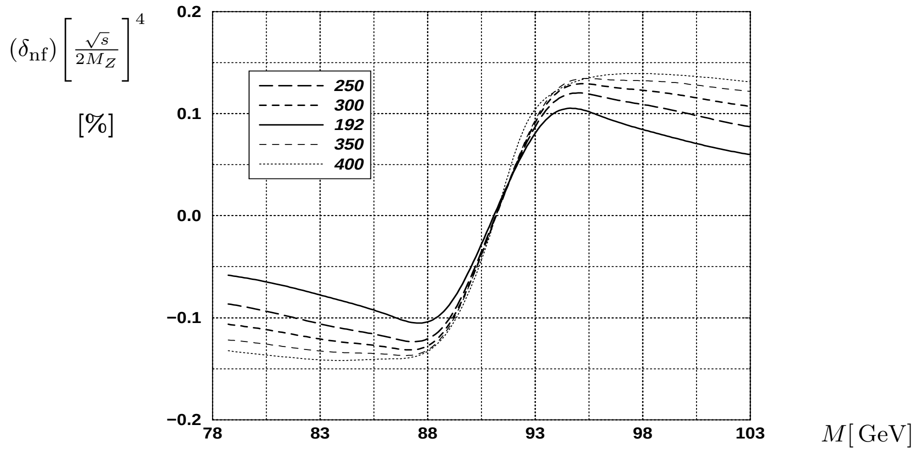


Fig. 4. The nonfactorizable correction to the distribution over the Z-boson invariant mass in $e^+e^- \to ZZ \to d\bar{d}u\bar{u}$. In order to elucidate the role of the screening factor, the nonfactorizable correction, $\delta_{\rm nf}$, is multiplied by the $(\sqrt{s}/2M_Z)^4$ factor. The curves are given for $\sqrt{s} = 192$, 250, 300, 350 and 400 GeV

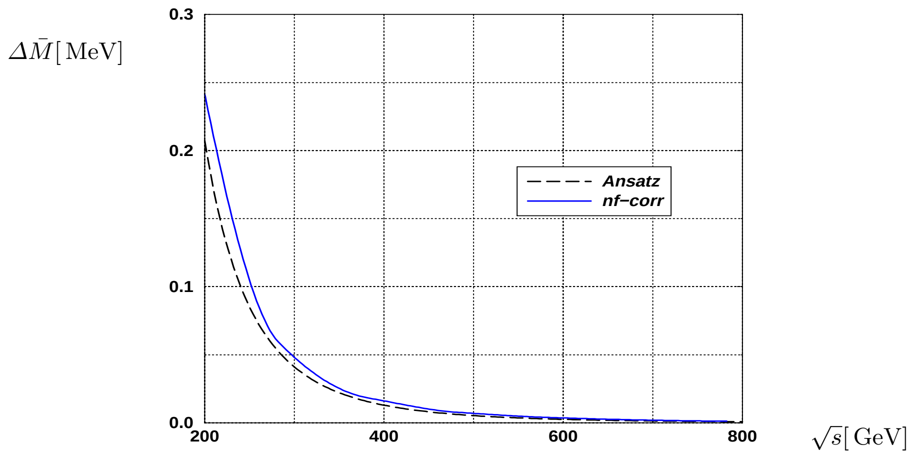


Fig. 5. The additional shift of the maximum in the Z-boson invariant mass distribution due to the nonfactorizable correction as a function of the collider energy, as compared to the expectations from the screened-ansatz scenario: $\delta_{\rm nf}^{ZZ} \sim (2M_Z/\sqrt{s})^4$

Coulomb radiative interferences on the width-dependent effects. It should be stressed that throughout the paper, the nonfactorizable corrections are calculated for the purely leptonic final state (for example, $\mu^+\nu_\mu e^-\bar{\nu}_e$). Strictly speaking, outside threshold region, nonfactorizable corrections to other final states (i.e., semi-hadronic and purely hadronic final states) are not identical [11], but the differences are, in fact, not so large.

Figure 1 compares the distribution over the average invariant mass $\bar{M} = (M_1 + M_2)/2$ in the three scenarios above at $\sqrt{s} = 172$, 183, and 195 GeV. Figure 2 shows the additional mass shift $\Delta\bar{M}$, from the nonfactorizable effects, as a function of the collider energy. The expectation corresponding to (11) is also shown. One can see a remarkable agreement between the result of the complete calculations and a simple screening recipe (11) for the mass shift.

For practical purposes, it is useful to analyze the impact of the instability effects on W-momentum distribution; see, e.g., [8]. Figure 3 compares the results for the differential momentum distribution $\mathrm{d}\sigma/\mathrm{d}p$ in the three scenarios at $\sqrt{s} = 172$, 183, and 195 GeV.

Figures 1 and 3 clearly show the dampening role of the screening factor $(1-\beta)^2$. In particular, a sharp increase in $\delta_{\rm nf}$ around $p = p_0 = \sqrt{EM_W}$ becomes much less pronounced as compared to the unscreened case; see also [8]. The plots demonstrate that the screened-Coulomb ansatz is quite reliable, even for momenta that significantly deviate from p_0.

Finally, recall that the high-energy behaviour of the nonfactorizable corrections to the ZZ production is of special interest, in particular because of a certain resemblance between these QED interference effects and colour-interconnection phenomena in the gauge-boson pair production; see, e.g., [17]. An explicit numerical calculation confirms the presence of the same screening $(1-\beta)^2$ factor in this case; see Figs. 4 and 5. As in to the WW case, this factor also has its origin in the conservation of currents.

4 Conclusions

The success of the precision studies of W-boson physics relies on an accurate theoretical knowledge of the details of the production and decay mechanisms. The instability of the W bosons can, in principle, strongly modify the standard stable-W results. An important role can be played by the radiative interference effects, which prevent the final state in $e^+e^- \to W^+W^- \to 4$ fermions from being treated as two separate W decays. Thus, purely QED interaction between two unstable W bosons induces nonfactorizable corrections to various final-state distributions. The complete analytical calculations of these corrections have been performed only recently [9–11]. In this paper, we demonstrate explicitly that a simple, physically motivated ansatz allows one to approximate the nonfactorizable corrections to the single-inclusive final state distributions with a surprisingly good accuracy. This approach makes the physical insight into the effects of instability of the W-pair production quite transparent.

One has to bear in mind that, typically, the order of magnitude of the nonfactorizable corrections does not exceed 1%, and that their practical relevance strongly depends on the requirements of the experiment; in particular, they could match the expected accuracy of measurements at a future lepton collider.

Finally, let us note that a similar screening scenario could, in principle, provide a useful framework for studies of QCD final-state interactions in $e^+e^- \to t\bar{t}$ and of the colour-interconnection effects in W^+W^- production (see, e.g., [17]). For example, it could be explicitly checked that the result of the calculation of one-loop QCD interconnection effects in $t\bar{t}$ production is consistent with the ansatz proposed in this paper.

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Appendix
Screening effects: quantitative discussion

The aim of this appendix is to expose the origin of the screening $(1-\beta)^2$ factor, based on the explicit evaluation of Feynman diagrams.

We first consider a model case [4] (see also [7]) in which one of the W bosons is assumed to be stable, and the other one has the standard decay modes with decay width Γ_W. In this way, we can gain insight into the analytical structure of the interference effects without encountering the complications which occur in the case of two unstable bosons. Our final result for the nonfactorizable corrections in such a hypothetical case fully agrees with that in [4]. Nevertheless, we find it instructive to present our alternative (more transparent) derivation. The results obtained within this simpler model will be used in the discussion of the realistic process in which both W bosons are off-shell.

In this model example, the virtual nonfactorizable correction can be written as the interference between two currents:

$$\mathrm{d}\sigma_{\mathrm{nf}} = \mathrm{d}\sigma_{\mathrm{Born}} 2\mathrm{Re}\, 4\pi\alpha\mathrm{i} \int \frac{\mathrm{d}^4k}{(2\pi)^4 k^2} \frac{p_2^\mu}{(-p_2k)} \times \left[\frac{p_1^\mu}{p_1k} - \frac{k_1^\mu}{k_1k}\right] \frac{D_1}{D_1 + 2p_1k}, \tag{A.1}$$

where

$$D_1 = p_1^2 - M_W^2 + \mathrm{i}M_W\Gamma_W, \tag{A.2}$$

p_1 and p_2 are the 4-momenta of the unstable W^- boson and stable W^+ boson, respectively, and k_1 and k_1' are the 4-momenta of the decay products of the off-shell W boson, $p_1 = k_1 + k_1'$.

Note that here and in what follows, the k^2 terms in the propagators of the radiating particles are neglected. Outside the threshold domain, an account of these terms gives a negligibly small (order Γ_W/E) effect.

We must also include the corresponding (A.1) contribution coming from the real-photon radiatiative interference. Note that throughout this paper, we do not consider the nonfactorizable corrections involving the initial-state radiation because of the cancellation between the virtual and real pieces (for details, see, e.g., the first reference in [2]. This also allows one to apply the results to the photon-photon initiated process.

The integrand in (A.1) has two poles in the upper-half k^0 plane (the photon pole and the W^- pole). The remaining poles are located in the lower-half plane. When we perform the $\mathrm{d}k^0$ integration by closing the contour in the upper-half-plane, we see immediately that the contributions from the real and virtual pieces cancel each other. This exemplifies the well-known cancellation between the real and virtual emissions.[3] Thus, the only nonzero contribution comes from the W^- pole. We found it especially convenient to perform the analysis of this contribution in the rest frame of the W^-. In this Lorentz frame, the particle 4-momenta can be written as: $p_1^\mu = (E_1; \mathbf{p}_1)$, $|\mathbf{p}_1| = \tilde{\beta}E_1$, $p_2^\mu = (M_W, \mathbf{0})$, $k_1^\mu = (\epsilon_1; \mathbf{k}_1)$

Let us now evaluate the integral

$$I = \mathrm{Re}\,\mathrm{i} \int \frac{\mathrm{d}^4k}{k^2} \mathrm{Pole}^{\mathrm{up}} \frac{(p_2k_1)}{(-p_2k)(k_2k)} \frac{D_1}{D_1 + 2p_1k}. \tag{A.3}$$

"$\mathrm{Pole}^{\mathrm{up}}$" denotes that the residue should be taken in the poles located in the upper-half plane, thus only the $(-p_2k)$ pole contributes. In the W^- rest frame this term is just $-p_2k = -\omega M_W + \mathrm{i}o$. Let us take the residue of this pole and use the cylindrical coordinates

$$I = -\mathrm{Re}\, 2\pi\epsilon_1 \int \frac{\mathrm{d}k_{||} k_\perp \mathrm{d}k_\perp \mathrm{d}\phi}{k_{||}^2 + k_\perp^2} \frac{1}{(-k_{||}k_{1||} - \mathbf{k}_\perp \mathbf{k}_{1\perp} + \mathrm{i}o)} \times \frac{D_1}{D_1 - 2|\mathbf{p}|k_{||}}. \tag{A.4}$$

Now we carry out the $k_{||}$ integration.The integrand has radiating particle poles, which are located in the upper-half $k_{||}$ plane. The photon poles occur at $k_{||} = \pm\mathrm{i}k_\perp$. Now we close the contour in the lower-half plane in order to avoid all charged-particle poles. Then the interference contribution becomes

$$I = -\mathrm{Re}\, 2\pi^2\epsilon_1 \int \frac{\mathrm{d}k_\perp \mathrm{d}\phi}{k_\perp} \frac{1}{(\mathrm{i}k_{1||} - |\mathbf{k}_{1\perp}|\cos\phi)} \times \frac{D_1}{D_1 + 2|\mathbf{p}|\mathrm{i}k_\perp}. \tag{A.5}$$

[3] This cancellation is similar to the case of the nonfactorizable corrections caused by the initial–final state radiative interference, see [2,4].

The integration over the azimuthal angle ϕ is quite straightforward. The integral over $k_\perp$ is infrared-divergent, but the divergent piece is purely imaginary. Finally, we arrive at

$$I \sim \frac{\epsilon_1}{|\mathbf{k_1}|} \mathrm{Re\,i\,ln}\,\frac{D_1}{\mathrm{i}}. \tag{A.6}$$

So far, we have evaluated only the second term in (A.1). The first term can be treated in an analogous way after the substitution $k_1 \to p_1$. The complete nonfactorizable correction is then given by

$$\delta_{\mathrm{nf}} \sim \left(\frac{E}{|\mathbf{p}_1|} - \frac{\epsilon_1}{|\mathbf{k_1}|}\right)\mathrm{Re\,i\,ln}\,\frac{D_1}{\mathrm{i}} = \frac{1-\tilde{\beta}}{\tilde{\beta}}\mathrm{Re\,i\,ln}\,\frac{D_1}{\mathrm{i}}. \tag{A.7}$$

The fact that the prelogarithmic factor approaches zero as $\tilde{\beta} \to 1$ is a direct consequence of the charged-current conservation. Recall that $\tilde{\beta} = \sqrt{1 - M_W^2/E_1^2}$ is taken in the system where W^- is at rest. Now we return to the center-of-mass frame, where the velocities of W is β ($\beta = \sqrt{1 - 4M_W^2/s}$); it is connected to $\tilde{\beta}$ by $\tilde{\beta} = 2\beta/(1+\beta^2)$. This allows us to present the complete nonfactorizable correction in the canonical ansatz form

$$\delta_{\mathrm{nf}} \sim \frac{(1-\beta)^2}{2\beta}\arctan\frac{M_1^2 - M_W^2}{M_W \Gamma_W}. \tag{A.8}$$

Now we turn to the realistic case of two off-shell W bosons. We shall concentrate on the high-energy behaviour of the complete nonfactorizable correction.

The virtual nonfactorizable correction can be presented in a standard form as a sum of the current interferences

$$\mathcal{M}_{\mathrm{nf}}^{\mathrm{virt}} = \mathrm{i}\mathcal{M}_{\mathrm{Born}} \int \frac{\mathrm{d}^4 k}{(2\pi)^4 k^2}\left[(\mathcal{J}_0\mathcal{J}_+) + (\mathcal{J}_0\mathcal{J}_-) + (\mathcal{J}_+\mathcal{J}_-)\right]. \tag{A.9}$$

The currents are given by

$$\begin{aligned}
\mathcal{J}_0^\mu &= \mathrm{e}\left[\frac{p_1^\mu}{kp_1} + \frac{p_2^\mu}{-kp_2}\right],\\
\mathcal{J}_+^\mu &= -\,\mathrm{e}\left[\frac{p_1^\mu}{kp_1} - \frac{k_1^\mu}{kk_1}\right]\frac{D_1}{D_1 + 2kp_1},\\
\mathcal{J}_-^\mu &= -\,\mathrm{e}\left[\frac{p_2^\mu}{-kp_2} - \frac{k_2^\mu}{-kk_2}\right]\frac{D_2}{D_2 - 2kp_2}
\end{aligned} \tag{A.10}$$

Here $p_{1,2}$ are the 4- momenta of the W bosons, and $k_{1,2}$ are the 4-momenta of the corresponding charged-decay products. There is also a corresponding contribution from the real-photon radiation interferences. The first and the second terms in (A.9) can be treated in exactly the same way as the previous model case. Therefore, we shall concentrate on the third term, which has a different analytical structure.

Recall that at higher energies, the dominant contribution to the radiative interference effects comes from the photons (real or virtual), with the energies

$$\omega \sim \Gamma_W \frac{M_W}{E_W}, \quad 2E_W = \sqrt{s}; \tag{A.11}$$

see, e.g., [13]. One can arrive at the same conclusion from an explicit estimate of the dominant contribution to the integral (A.10).

To be specific, we concentrate below on the typical case of the W-decay mass distribution. Therefore, it is assumed that the integration over the decay products has already been carried out. Let us analyze the consequences of this integration for the Born decay cross section and for the nonfactorizable currents (A.10). First, recall that the squared Born decay matrix element can be written as

$$\begin{aligned}
\mathcal{M}_{\mathrm{dec}}^\mu \mathcal{M}_{\mathrm{dec}}^{*\,\nu} &\sim \frac{1}{4}\mathrm{Sp}\left[\gamma^\mu(1-\gamma^5)\ \not{k}_1\gamma^\nu(\not{p}_1 - \not{k}_1)\right]\\
&= \Delta_V^{\mu\nu} - \mathrm{i}\Delta_A^{\mu\nu},\\
\Delta_V^{\mu\nu} &= k_1^\mu p_1^\nu + k_1^\nu p_1^\mu - 2k_1^\mu k_1^\nu - g^{\mu\nu}\frac{M_W^2}{2},\\
\Delta_A^{\mu\nu} &= \epsilon^{\mu\nu k_1 p_1},
\end{aligned} \tag{A.12}$$

where indices μ and ν are to be contracted with the corresponding ones in the production part of the Born cross section. We use the notation $\epsilon^{\mu\nu\rho p} = \epsilon^{\mu\nu\rho\alpha}p_\alpha$. Let us start from the vector piece $\Delta_V^{\mu\nu}$.

The Born decay cross section, integrated over the phase space of the decay products, is given by

$$\begin{aligned}
\mathcal{I}_{\mathrm{Born}}^{\mu\nu} &= \int \mathrm{d}^4 k_1\ \delta(k_1^2)\ \delta\big(M_W^2 - 2(p_1 \times k_1)\big)\ \times\ \Delta_V^{\mu\nu}\\
&= \frac{\pi}{6}M_W^2\left[g^{\mu\nu} - \frac{p_1^\mu p_1^\nu}{M_W^2}\right].
\end{aligned} \tag{A.13}$$

Consider now an integral over the corresponding nonfactorizable current

$$\begin{aligned}
\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha} &= \int \mathrm{d}^4 k_1\ \delta(k_1^2)\ \delta\big(M_W^2 - 2(p_1 \times k_1)\big)\ \times\ \Delta_V^{\mu\nu}\\
&\quad \times \left[\frac{p_1^\alpha}{kp_1} - \frac{k_1^\alpha}{kk_1}\right].
\end{aligned} \tag{A.14}$$

Tensor $\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha}$ can depend only on the 4-vectors p_1^μ and k^μ, and has the following general features:

- $\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha} = \mathcal{I}_{\mathrm{nf},\ V}^{\nu\mu\alpha}$,
- $\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha} p_{1,\ \mu} = 0$,
- $\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha} k_\alpha = 0$,
- $\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha} g_{\mu\alpha} = 0$. (A.15)

It is convenient to carry out the integration in the W-boson rest frame. The integral $\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha}$ simplifies in the high-energy limit if one recalls that only the soft photons (A.11) are responsible for the nonfactorizable correction. Then $p_1^\mu \sim E_W$, and $k^\mu \sim \Gamma_W M_W/E_W$.

$$\begin{aligned}
\mathcal{I}_{\mathrm{nf},\ V}^{\mu\nu\alpha} &= A\left(\frac{p_1^\alpha}{kp_1} - \frac{k^\alpha}{k^2}\right)\Bigg[g^{\mu\nu} + \frac{k^2}{(kp_1)^2}p_1^\mu p_1^\nu + M_W^2\frac{k^\mu k^\nu}{(kp_1)^2}\\
&\quad - \frac{1}{(kp_1)}\left[k^\mu p_1^\nu + p_1^\mu k^\nu\right]\Bigg],
\end{aligned} \tag{A.16}$$

$$A = \frac{\pi}{4} \frac{M_W^4 k^2}{(kp_1)^2} \left[1 + \frac{1}{2} \ln \frac{M_W^2 k^2}{4(kp_1)^2} \right]. \qquad \text{(A.17)}$$

Note that $A \sim E_W^{-2}$ (we keep track of the energy dependence only), since $(kp_1) \sim M_W \Gamma_W$ and $k^2 \sim \Gamma_W^2 M_W^2 / E_W^2$. Recall also that the $\mu\nu$ tensor in the equation above is of the same or lower order as in the Born approximation; see (A.13).

Now we can readily obtain an upper limit for the third term in the square brackets in the integrand in (A.9).

$$\begin{aligned}
\mathcal{M}^{\text{virt}}_{\text{nf, V}} \sim & \int \mathrm{d}^4 k_1 \ \delta(k_1^2)\ \delta\big(M_W^2 - 2(p_1 \cdot k_1)\big) \\
& \times \Delta_V^{\mu\nu}(p_1, k_1) \\
& \times \int \mathrm{d}^4 k_2 \ \delta(k_2^2)\ \delta\big(M_W^2 - 2(p_2 \cdot k_2)\big) \\
& \times \Delta_V^{\mu'\nu'}(p_2, k_2) \\
& \times \int \frac{\mathrm{d}^4 k}{(2\pi)^4 k^2} (\mathcal{J}_+ \mathcal{J}_-) \\
\lesssim & \left[g^{\mu\nu} - \frac{p_1^\mu p_1^\nu}{M_W^2} \right] \left[g^{\mu\nu} - \frac{p_2^{\mu'} p_2^{\nu'}}{M_W^2} \right] \\
& \times \frac{E_W^{-4}}{E_W^{-2}} \cdot E_W^{-2} E_W \cdot E_W^{-2} E_W. \qquad \text{(A.18)}
\end{aligned}$$

As a result, the nonfactorizable correction is shown to acquire, at high energies, an additional (screening) factor

$$\delta_{\text{nf}} \sim \frac{1}{E_W^4} \sim (1-\beta)^2. \qquad \text{(A.19)}$$

The E^{-4} screening effect can be roughly understood as follows: Two powers of energy result from the photon phase space, and the other two come from the two interfering nonfactorizable currents, one power of energy from each current.

To make the consideration complete, we turn now to the axial piece of the Born decay cross section, the $\Delta_A^{\mu\nu}$ term in (A.12). It is possible to treat this contribution in an analogous way as before. Here we present the result of the integration over the decay phase space,

$$\begin{aligned}
\mathcal{I}^{\mu\nu\alpha}_{\text{nf, }A} = & \int \mathrm{d}^4 k_1 \ \delta(k_1^2)\ \delta\big(M_W^2 - 2(p_1 \cdot k_1)\big) \\
& \times \epsilon^{\mu\nu k_1 p_1} \ \times \left[\frac{p_1^\alpha}{kp_1} - \frac{k_1^\alpha}{kk_1} \right] \\
= & B \left[k^\alpha \epsilon^{\mu\nu k p_1} - k^2 \epsilon^{\mu\nu\alpha p_1} \right] \\
& + C \left[p_1^\alpha \epsilon^{\mu\nu k p_1} - (kp_1) \epsilon^{\mu\nu\alpha p_1} \right], \qquad \text{(A.20)}
\end{aligned}$$

where

$$\begin{aligned}
B &= -\frac{\pi}{8} \frac{M_W^4}{(kp_1)^3} \left[1 + \ln \frac{M_W^2 k^2}{4(kp_1)^2} \right], \\
C &= +\frac{\pi}{8} \frac{M_W^2}{(kp_1)^2} \left[1 + \frac{3k^2 M_W^2}{2(kp_1)^2} \ln \frac{M_W^2 k^2}{4(kp_1)^2} \right]. \qquad \text{(A.21)}
\end{aligned}$$

Note that $B \sim E^0$ and $C \sim E^0$ at high energy (again we keep track of the energy dependence only). However, the Lorentz structure of (A.20) is different from that in the Born approximation. A nonfactorizable current integrated over the angles of the decay products $\mathcal{I}^{\mu\nu\alpha}_{\text{nf, }A}$ is supressed by one power of energy, as compared to the Born approximation, $\sim E_W^{-1}$. An estimate similar to (A.18) shows that the relative nonfactorizable correction behaves as $\sim E_W^{-4}$, where half of the supression comes from the phase space of the photon, $\sim E_W^{-2}$, and the other half comes from the interference between two nonfactorizble currents: about E_W^{-1} from each one. Thus, the result (A.19) remains valid when the axial contribution is taken into account.

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